Steady-State Drainage of an Aqueous Foam

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The process of wetting of a monodisperse foam by the continuous addition of liquid from the top has been investigated for a cylindrical column. The velocity of the interface between dry and wet foam was found to vary as the square root of the rate of addition of liquid. Evidence of a structural transformation was found around gas fraction $\Phi=0.87$. It was interpreted as arising from the collapse of the four-sided faces of the (Kelvin) polyhedra which constitute the foam cells, and this was directly observed when large bubbles were used. There is a second transition at around $\Phi=0.6$, with the onset of bubble motion.

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The structure of a liquid foam, and its relation to the characteristic properties of this complex two-phase system, present a classic problem which has received fresh attention in a number of computational and experimental studies, mainly for two-dimensional systems [1]. In particular, progress has been made in understanding the coarsening process, due to diffusion of gas between cells. The explanation of remaining anomalies in coarsening data has focused on the role of Plateau borders [2], the liquid-filled regions at the intersection of cell walls (Fig. 1), often ignored in studies of "dry" foam with a small liquid fraction. In turning to the case of a three-dimensional foam, an additional complication presents itself, in the vertical drainage of the liquid under gravity. The underlying mechanisms of this key aspect of foam behavior were addressed in the definitive work of Mysels, Shinoda, and Frankel [3], mainly concerned with the thinning of a single soap film, but they remain debatable. More recently Princen [4] has discussed the vertical equilibrium profile towards which the draining system must tend; this is in itself an awkward problem.

There seem to be relatively few helpful experiments. The obvious one, monitoring drained liquid as a function of time, does not offer much immediate insight [5]. Alternatively, if liquid is fed in at the top of the system so that a state of steady drainage rather than that of equilibrium is approached, it turns out to result in more easily interpreted phenomena. The only previous related experiments of which we are aware are those of Noever and Cronise [6] on two-dimensional systems. Surprisingly, the initial phase in which a dry foam is being progressively wetted exhibits very simple behavior, conforming to a power law which can be rationalized. As the flow rate is increased, two distinct anomalies are encountered, attributable, respectively, to structural changes and the onset of motion of bubbles.

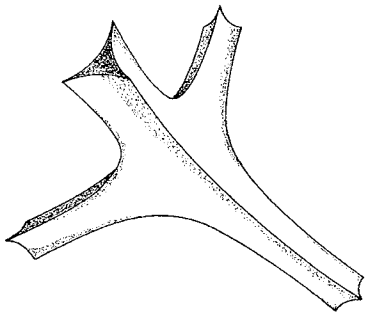

FIG. 1. Plateau borders, meeting at a fourfold junction.

The experimental procedure is elementary. A tube is placed vertically in a bath of detergent solution, as indicated in Fig. 2. A foam is created within the tube, usually by blowing bubbles of N_2 through a fine nozzle placed beneath it. When performed at a constant slow rate, this creates a foam of uniform cell size, and hence an ordered structure. A more typical, disordered foam may be made by blowing the gas through a filter. Once the foam has drained to a state close to its equilibrium (in a few minutes), further solution is introduced from the top at a steady rate.

The first, and striking, observation is that the boundary between the dry and wetted foam remains very sharp, as in the 2D observations of Noever and Cronise [6], so that its velocity is easily measured. Moreover, this velocity remains constant as wetting proceeds. It was recorded for a series of different flow rates, yielding results as in Fig. 3(a). Also shown, in Fig. 3(b), is the variation of gas fraction Φ, estimated simply by noting the height of wet foam created, plotted against the volume flow rate Q of the solution. These quantities are approximately related by

$$Q=(1-\Phi)Av\,, \tag{1}$$

where A is the cross-sectional area of the tube. (This neglects the liquid content of the dry phase: Note also that the top surface of the wetted foam does not move significantly.) The data of Figs. 3(a) and 3(b) are con-

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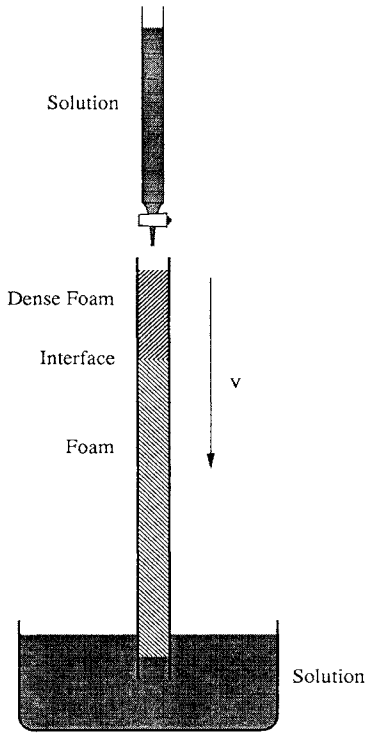

FIG. 2. Schematic experimental arrangement.

sistent with such a relation.

Both dependences on flow rate are well described by a square-root dependence, $v \sim Q^{1/2}$, $(1-\Phi) \sim Q^{1/2}$. Values of the power-law index obtained by curve fitting give a mean of 0.53 ± 0.07 for nine runs under different conditions (e.g., different bubble sizes, always much less than that of the tube.)

This square-root dependence may be explained as follows. We expect the essential mechanism of flow over most of the accessible range to be the longitudinal motion of Plateau borders. These form a network of "pipes," as in Fig. 1, not with the fixed boundary conditions that would dictate Poiseuille flow, but rather with free boundary conditions. Nevertheless the flow requires shearing motion, and the flow rate in each pipe should show the same dependence on the *square* of cross-sectional area as in Poiseuille flow, on dimensional grounds. To lowest order in $1-\Phi$ this gives $Q \sim (1-\Phi)^2$, in keeping with our results. However, it is clear that this kind of scaling argument can only be valid for small $1-\Phi$, since the geometry and topology of the network itself must alter significantly as the liquid fraction increases. Indeed, we shall see that the data reflect such changes, when examined closely. Furthermore, this drainage mechanism is not unique. Alternatively, motion of the cell walls into and out of the borders can contribute [3].

In some cases, the simple picture of flow with free boundary conditions may be inadequate. For some surfactants a large surface viscosity must be considered.

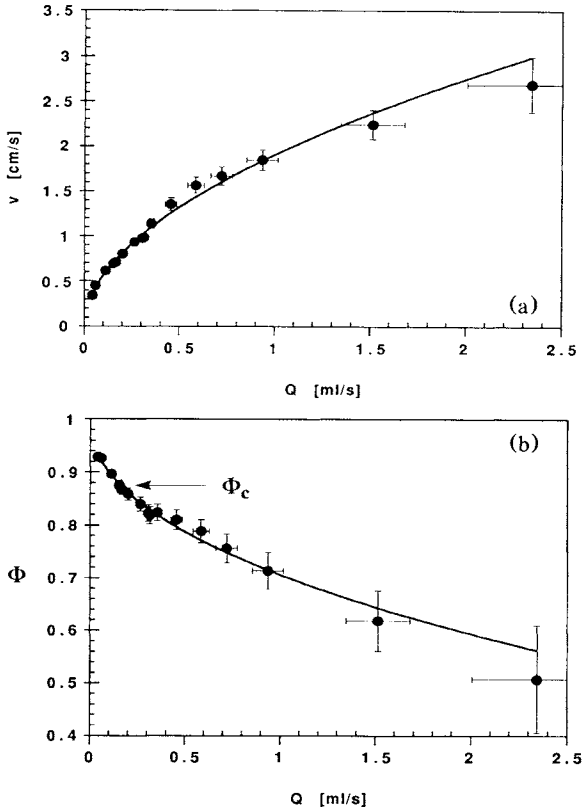

FIG. 3. Data for a foam made up of bubbles of diameter 0.77 mm, in a tube of diameter 15 mm. Height of foam column is approximately 350 mm. (a) Interface velocity as a function of flow rate Q. The fitted curve is $v \sim Q^{\alpha}$, where $\alpha = 0.53$. (b) Liquid fraction $(1-\Phi)$ as a function of Q. The fitted curve is $(1-\Phi) \sim Q^{\beta}$, where $\beta = 0.47$ [$\alpha+\beta = 1.0$, in accord with Eq. (1)]. Structural rearrangements were observed mainly at $\Phi_c = 0.88$ in this case.

However, we believe that this is not the case for the detergent used here, and the success of the dimensional argument (based on bulk viscosity) supports this.

This simple experimental arrangement provides us with an easily controlled variable gas fraction between 1 and about 0.5, which should be very convenient for a variety of further measurements, some of which may be stimulated by the following observations. For low flow rates (and hence high Φ), the liquid drains between the cells, and the cells retain their positions, more or less. This scenario cannot be sustained at large flow rates. The cells must eventually separate as isolated bubbles, so that the foam loses stability. This is essentially the "rigidity loss transition" described by Bolton and Weaire [2], but this has been theoretically analyzed only for 2D (static) models [2].

At high flow rates bubbles are thus free to move in convective motion. We have found it difficult to assign a precise value of Φ to this instability: It was generally observed in the range 0.50–0.65. This is to be compared

with the critical density $\Phi=0.74$, at which the rigidity loss transition of a close-packed equilibrium structure is expected in three dimensions. Thus the loss of mechanical stability of the corresponding static system does not immediately lead to large-scale bubble motion.

Another anomaly, which was not anticipated, showed up in the finer detail of the v/Q relationship. A small departure from the smooth square-root variation of v was repeatedly observed when the gas fraction passed through the range $\Phi=0.8$–0.9. We believe that the explanation of this is to be found in the foam structure, as follows.

In contrast to the situation in two dimensions, the equilibrium structure of an ordered 3D foam is not self-evident. For the case of a dry foam, the assertion of Lord Kelvin [7], that the body-centered cubic arrangement of cell centers is optimal, remains the conventional wisdom [8]. The corresponding cell shape is sketched in Fig. 4. However, in the opposite limit of low gas fraction (dry foam), it is clear that close-packed structures such as fcc must have lower energy: Hence there ought to be at least one structural transformation, as gas fraction is decreased. Even if we retain the bcc structure, there is a critical value of Φ, at which second-neighbor cells lose contact (see Fig. 4). This in itself could lead to an anomaly in drainage, but it may well provoke a larger structural change. It is a familiar property of pairwise-potential models of crystal structure that the bcc case is unstable for nearest neighbor interactions, and contacting bubbles may be closely analogous. The collapse of four-sided faces corresponds to the loss of contact between second-nearest neighbors. It is also true that a junction of more than four borders is unstable for the dry foam, but this is not necessarily the case for finite liquid fraction.

We saw evidence of structural change in visible disturbances of cell positions, within the above range of Φ. From a large number of such observations we estimate the critical value to be $\Phi_c=0.87\pm0.04$. However, one is frustrated by the difficulty of simple visual observation. This is because the surface structure in the tube is pinned in a close-packed structure, and only the central core constitutes the bulk to which such arguments apply.

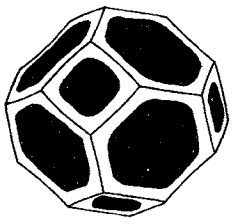

FIG. 4. In the structure proposed by Kelvin, Plateau borders are arranged as shown around the (slightly curved) faces of a polyhedral cell (minimal tetrakaidecahedron). The shaded areas indicate the contacts between adjoining bubbles, while the white areas are sections through the Plateau borders.

In order to provide further evidence for this structural change, we prepared samples with relatively large bubbles, comparable in size to the diameter of the tube. We have previously pointed out [9] that a sequence of beautiful ordered structures is readily formed in such a case. For a suitable bubble size, the central core is a single string of Kelvin polyhedra, as in Fig. 4. In this system we may closely observe the effects of increasing the flow rate Q, since the entire structure is clearly visible. As the contacts on the four-sided faces are eliminated, there is a rapid sequence of topological changes, resulting in a transformation of the entire structure. The new structure is again periodic, but always lacks the four-sided faces which provoke the original instability. In at least some cases, its core is made up of pentagonal dodecahedra.

The critical value of Φ which we have identified for the structural change also compares well with a crude estimate of the expected value for the closing of the four-sided cell faces, as follows. For low liquid fraction, $1-\Phi$ should scale as the square of the transverse dimension of the Plateau borders. That is, each Plateau border makes a contribution proportional to its length and cross-sectional area, the latter being treated as constant along its length. The obvious corrections to this (overlap at and shape of the junctions) appear to largely cancel, as indicated by the fact that an alternative estimate based on inscribing a sphere to touch the fourfold faces gives much the same result.

The ratio of the liquid fractions necessary to create Plateau borders whose width is such as to span the four- and six-sided faces is 1:3, which is simply the ratio of the widths of these faces. Since the loss of contact on the six-sided faces corresponds to the separation of bubbles, for which Φ is 0.68 (packing fraction of bcc, the resulting estimate for the liquid fraction for the collapse of the four-sided cell faces is $\Phi=1-(1-0.68)/3=0.89$.

Any more complete analysis of structural stability, whether for ordered or disordered structures, should also take account of the shear stress resulting from the compression-expansion of the foam within a fixed cross section.

In previous descriptions of foam structure, such as that of Dormer [10], it has often been claimed that the Kelvin polyhedron, while acceptable in principle, does not predominate in practice, even for cells of equal size (monodisperse foam). This should now be reconsidered: Direct experimental measurements of three-dimensional foam structures are needed.

In conclusion, we have demonstrated that steady-state drainage, in the sense defined above, offers new opportunities for foam experiments, and exhibits evidence of a hitherto unobserved structural transformation, as well as a transition to a different type of flow. On closer examination, it should provide new insights into the drainage process itself.

Extensions of this work which are still in progress

include the formulation of a mathematical theory of drainage [11], which exhibits the qualitative features reported here, and a more complete analysis of the stability of three-dimensional foam structures [12].

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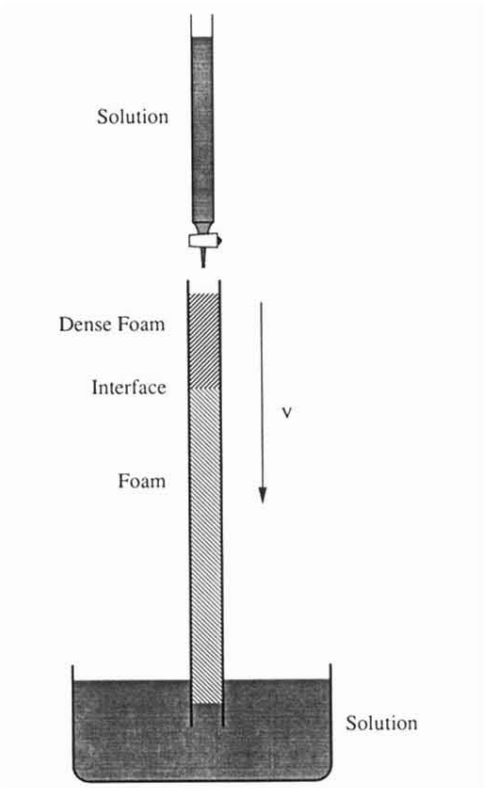

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