Theoretical Foundations of Chemical Engineering, Vol. 39, No. 3, 2005, pp. 329–331.
Translated from Teoreticheskie Osnovy Khimicheskoi Tekhnologii, Vol. 39, No. 3, 2005, pp. 349–351.

SHORT COMMUNICATIONS

Quantitative Determination of the Limits of Hydrodynamic Modes of Flotation in Rheologically Complex Media

O. M. Sokovnin and N. V. Zagoskina

Vyatka State University, ul. Moskovskaya 36, Kirov, 610000 Russia
e-mail: osokovnin@mail.ru

Received August 19, 2003

Abstract—The introduction of a new parameter—the modified Stokes number—is proposed and substantiated. This parameter quantitatively determines the limits of the modes of inertial and inertialess flotation in rheologically complex media with non-Newtonian viscosity.

Flotation is based on the physical effect of sedimentation and adhesion of suspended particles to the surface of a rising bubble. Flotation technologies are widely used to separate liquid inhomogeneous systems. In this case, the dispersed phase (solid particles in suspensions and liquid drops in emulsions) are separated from the dispersion medium. Flotation is conventionally used for ore dressing. This is the so-called inertial flotation, which is characterized by high concentrations (up to hundreds of kilograms per cubic meter of flotation medium), density ($\rho_p \gg \rho_l$), and sizes ($d_p \ge d_b$) of separated particles. In addition, in recent decades, flotation has been used in microbiology, the food industry, and water treatment. This is the so-called inertialess flotation that is characterized by low concentrations (as low as several grams per cubic meter of flotation medium) of separated micronic and submicronic particles ($d_p \ll d_b$), whose density ρ_p is close to the density ρ_l of the flotation medium. Examples of such flotation include the concentration of cultural media in microbiology, juice settling, and wastewater treatment to remove oils, fats, and emulsified petroleum products [1–3].

A quantitative measure of the limits of the modes of inertial and inertialess flotation is the Stokes number

$$\mathrm{St} = d_p^2 W(\rho_p - \rho_l)/18\mu_l d_b, \tag{1}$$

where W is the relative velocity of the bubble and the particle.

The following hydrodynamic modes of flotation are distinguished [4]:

(i) at $\mathrm{St} \le 0.01$, there is an inertialess mode, in which the inertial forces exert almost no effect on the particle moving along the streamlines in the vicinity of the bubble;

(ii) at $0.01 < \mathrm{St} \le 1$, there is a transient mode, in which the inertial and viscous forces are comparable;

(iii) at $\mathrm{St} > 1$, there is an inertial mode, in which the inertial forces are dominant in the interaction between the particle and the bubble and the trajectory of the particle approaching the bubble deviates slightly from a straight line.

These relations are valid for flotation media with constant (Newtonian) viscosity. However, the presence of dispersed-phase particles (fat or oil drops, high-molecular-weight compounds, surfactants, etc. [5]) in a flotation medium imparts certain non-Newtonian rheological properties to the medium. It is evident that such flotation media should also be described by a hydrodynamic parameter that is similar to expression (1) and that separates the regions of inertial and inertialess flotation.

Let us call this parameter the modified Stokes number $\tilde{\mathrm{St}}$ and determine its expression from the following reasoning. The Stokes number is the ratio of the inertial path length x_0 traveled by a particle with initial velocity W_0 in a continuous medium to the characteristic size l_0 [4]:

$$\tilde{\mathrm{St}} = x_0/l_0. \tag{2}$$

In the process under consideration (flotation), the characteristic size is the bubble size; i.e., $l_0 = d_b$. The differential equation of motion of a particle in a continuous medium in the steady-state mode has the form [6]

$$m_p W dW/dx = -CF_p\rho_l W^2/2. \tag{3}$$

The drag coefficient C in Eq. (3) for a non-Newtonian liquid in this hydrodynamic mode (at low Reynolds numbers) by analogy with an ordinary Newtonian liquid can be expressed as

$$C = 24/\tilde{\mathrm{Re}}. \tag{4}$$

For non-Newtonian flotation media, we use a two-parameter power-law model. Then, the modified Reynolds number in expression (4) can be written as [7]

$$\tilde{\mathrm{Re}} = \rho_l d_p^n W^{2-n}/K. \tag{5}$$

Substituting relations (4) and (5) into Eq. (3), expressing the weight and cross-sectional area of the particle through its diameter d_p and density ρ_p, and making certain transformations, we obtain

$$dx = -\frac{\rho_p d_p^{1+n}}{18K} W^{1-n} dW. \quad (6)$$

Integration of expression (6) with respect to x_0 from 0 to x_0 and with respect to W from W to 0 yields the desired value of the inertial path length in a continuous medium:

$$x_0 = \frac{\rho_p d_p^{1+n}}{18K(2-n)} W^{2-n}. \quad (7)$$

In this case, according to expression (2), the modified Stokes number for a non-Newtonian continuous medium is written as

$$\tilde{\mathrm{St}} = \frac{\rho_p d_p^{1+n}}{18K(2-n)d_b} W^{2-n}. \quad (8)$$

At $n = 1$, the modified Stokes number is identical to the Stokes number for a flotation medium with constant (Newtonian) viscosity ($K \equiv \mu$). The proposed expression for the modified Stokes number should be used as a quantitative parameter that separates the regions of inertial and inertialess flotation in rheologically complex media.

As an example, we determine the hydrodynamic mode of flotation separation of a fat-containing emulsion forming in vegetable oil refining in margarine production. We use the following starting data [8]:

the rheological constants of the fat-containing emulsion at working temperature ($t = 40°\mathrm{C}$) are 9.1×10^{-3} Pa s^n and $n = 0.60$;

the densities of the emulsion and fat particles are $\rho_l = 998$ kg/m^3 and $\rho_p = 920$ kg/m^3, respectively;

the size of flotation bubbles generated electrolytically at working current density ($i = 100$ A/m^2) is $d_b = 82$ μm;

the averaged (weighed over the specific area) size of fat particles is $d_p = 10.6$ μm;

the gas content of the flotation medium is $\varphi = 0.039$.

Because of the insignificant difference in density between the emulsified fat particles and the flotation medium, the relative velocity of the particle and the bubble can be taken to be equal to the rising velocity of the bubble: $W = W_b$. This velocity was determined in terms of the gas content φ of the non-Newtonian flotation medium by the formula [9]

$$W_b = \left[\frac{2\rho_l g}{3(3+2\varphi^{5/3})KI_1}\right]^{1/n} \times R_b^{(n+1)/n}(2 - 3\varphi^{1/3} + 3\varphi^{5/3} - 2\varphi^2). \quad (9)$$

Here,

$$I_1 = \int_0^{\pi}\left[2\cos^2\theta\left(\frac{9}{4} - 5\varphi^{5/3} + 5\varphi^{10/3}\right) + \sin^2\theta(36 + 12\varphi^{5/3} + \varphi^{10/3})\right]^{(n-1)/2}\sin\theta d\theta,$$

is an integral function, whose value was determined numerically [8] and, at $\varphi = 0.039$ and $n = 0.60$, is equal to 1.02. At $n = 1$, formula (9) appears as the relation derived previously [10] for a Newtonian liquid, and at $n = 1$ and $\varphi = 0$ we have the known Stokes formula for a single bubble with a stagnant surface.

Substituting the initial data into formula (9), we find $W_b = 1.76 \times 10^{-3}$ m/s. The numerical value of the modified Stokes number for the flotation system under consideration can be calculated from derived relation (8):

$$\tilde{\mathrm{St}} = \frac{920 \times (10.6 \times 10^{-6})^{1.6} \times (1.76 \times 10^{-3})^{1.4}}{18 \times 9.1 \times 10^{-3} \times (2 - 0.6) \times 82 \times 10^{-6}} = 7.43 \times 10^{-5}.$$

The result obtained shows that, in the system under consideration, there is a pronounced inertialess flotation mode: the inertial forces are much weaker than the viscous friction forces and the decisive factor in calculating the flotation separation efficiency is the long-range hydrodynamic interaction between the particle and the bubble.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project no. 03-01-00620.

NOTATION

C—drag coefficient;

d—diameter, m;

F—cross-sectional area, m^2;

g—acceleration of gravity, m/s^2;

K—rheological constant, a measure of consistence of a non-Newtonian liquid, Pa s^n;

m—weight, kg;

n—rheological constant, flow index of a non-Newtonian liquid;

R—radius, m;

W—velocity, m/s;

μ—dynamic viscosity, Pa s;

ρ—density, kg/m^3;

φ—gas content.

SUBSCRIPTS

b—bubble;

l—liquid;

p—particle.

REFERENCES

1. Ostermaier, K. and Dobias, B., The Separation of Proteins from Their Mixtures Using Flotation, *Colloids Surf.*, 1985, vol. 14, nos. 3–4, pp. 199–208.
2. Matov, B.M., *Flotatsiya v pishchevoi promyshlennosti* (Flotation in Food Industry), Moscow: Pishchevaya Promyshlennost', 1976.
3. Wastewater Treatment to Remove Petroleum Products, Fats, and Proteins: Basic Technologies, *Itogi Nauki Tekh., Ser.: Okhrana Prirody Vosproizvodstvo Prirodnykh Resursov*, Moscow: VINITI, 1988, vol. 20.
4. Dukhin, S.S, Rulev, N.N., and Dimitrov, D.S., *Koagulyatsiya i dinamika tonkikh plenok* (Coagulation and Dynamics of Thin Films), Kiev: Naukova Dumka, 1980.
5. Bibik, E.E., *Reologiya dispersnykh sistem* (Rheology of Disperse Systems), Leningrad: Leningr. Gos. Univ., 1981.
6. Romankov, P.G. and Kurochkina, M.I., *Gidromekhanicheskie protsessy khimicheskoi tekhnologii* (Hydromechanical Processes in Chemical Engineering), Leningrad: Khimiya, 1982.
7. Kutepov, A.M., Polyanin, A.D., Zapryanov, Z.D., Vyaz'min, A.V., and Kazenin, D.A., *Khimicheskaya gidrodinamika: Spravochnoe posobie* (Handbook of Chemical Hydrodynamics), Moscow: Byuro Kvantum, 1996.
8. Zagoskina, N.V., Calculation of Separation of Stable Fat-Containing Emulsions, *Cand. Sci. (Tech.) Dissertation,* Kazan: Kazan State Technol. Univ., 1997.
9. Sokovnin, O.M. and Zagoskina, N.V., Efficiency of the Entrainment of Rodlike Particles by Rising Bubbles, *Teor. Osn. Khim. Tekhnol.,* 2001, vol. 35, no. 3, pp. 316–320 [*Theor. Found. Chem. Eng.* (Engl. Transl.), vol. 35, no. 3, pp. 298–302].
10. Gal-or, B. and Waslo, S., Hydrodynamics of an Ensemble of Drops (or Bubbles) in the Presence or Absence of Surfactants, *Chem. Eng. Sci.*, 1968, vol. 23, pp. 1431–1446.