Theoretical Foundations of Chemical Engineering, Vol. 36, No. 6, 2002, pp. 601–603.
Translated from Teoreticheskie Osnovy Khimicheskoi Tekhnologii, Vol. 36, No. 6, 2002, pp. 658–660.

SHORT COMMUNICATIONS

Characterizing and Determining Particle Size

O. M. Sokovnin and N. V. Zagoskina
Vyatka State University, Moskovskaya ul. 36, Kirov, 610000 Russia
Received July 30, 2001

To design a mechanical or physicochemical separation process (sedimentation, filtration, centrifugation, flotation, wet dust catching, etc.) for a heterogeneous system such as a suspension, emulsion, or aerosol, it is necessary to know the particle-size composition of the dispersed phase. This characteristic is usually determined by sedimentation analysis, sieve analysis, microscopy, and other experimental methods [1, 2]. Particle-size analysis gives the weights of size fractions:

$$\eta_i = f(\bar{d}_i). \quad (1)$$

For the same system, the particle-size distribution can be determined experimentally in terms of sedimentation, linear, or projection diameters (by sedimentation, sieve, and microscopic analysis, respectively).

The size of suspended particles can be characterized by the linear diameter d_{10}, surface-area diameter d_{20}, and volumetric diameter d_{30}, which are calculated as

$$d_{10} = \frac{\sum_i (n_i \bar{d}_i)}{\sum_i n_i}, \quad (2)$$

$$d_{20} = \left(\frac{\sum_i (n_i \bar{d}_i^2)}{\sum_i n_i} \right)^{1/2}, \quad (3)$$

$$d_{30} = \left(\frac{\sum_i (n_i \bar{d}_i^3)}{\sum_i n_i} \right)^{1/3}. \quad (4)$$

Hence, the question arises as to which of the calculated diameters—d_{10}, d_{20}, or d_{30}—should be involved in an analysis of separation in heterogeneous systems and which of the experimental diameters—linear, projection, or sedimentation—should be used in calculations.

Particles are classified into the following three basic classes according to their shape [2, 3]:

(1) isometric particles (spheres, regular polyhedra), whose three dimensions are equal or similar;

(2) platelike particles (discs, plates), one of whose dimensions is much smaller than the other two; and

(3) needle-shaped particles, one of whose dimensions is much greater than the other two.

To answer the above questions, it is reasonable to determine the particle shape and then calculate d_{10} for needle-shaped particles, d_{20} for two-dimensional (platelike) particles, or d_{30} for three-dimensional particles. As a rule, the particle shape in polydisperse materials is determined by microscopy [2]. In the simplest cases, it can be deduced a priori from physical considerations. For example, it is obvious that emulsified oil and fat drops will always assume a spherical shape owing to surface tension.

Another aspect of the problem is selecting a method for particle-size analysis. For an adequate description of the particle-size distribution in a polydisperse material, it is advisable to use the method that is most similar in nature to the process under design. Sedimentation analysis is appropriate for determining the size of particles that are separated by sedimentation or centrifugation or are caught in wet gas cleaners. Sieve analysis is recommended for particles caught by filtration.

It is important to relate the observed particle-size distribution to the parameters calculated by formulas (2)–(4). These formulas are of little use, because the number of particles in the ith fraction, n_i, is difficult to determine. Let us express the calculated diameters in terms of the mean particle size and the weight fraction of the ith size fraction.

Obviously,

$$n_i = \frac{6m_i}{\pi\rho\bar{d}_i^3}. \quad (5)$$

By substituting Eq. (5) into Eq. (2), we obtain, after rearrangements,

$$d_{10} = \frac{\sum_i (m_i/\bar{d}_i^2)}{\sum_i (m_i/\bar{d}_i^3)}.$$

0040-5795/02/3606-0601$27.00 © 2002 MAIK "Nauka/Interperiodica"

Fractional makeup of fly ash

Material	Size fraction, μm							
	<4.0	4.0–6.3	6.3–10	10–16	16–25	25–40	40–63	>63
Coal ash	0.17	0.60	0.08	0.15	4.3	13.9	42.7	38.1
Turf ash	0.06	0.02	0.04	0.08	1.6	8.8	20.6	68.8
Coal–turf ash	0.10	0.35	0.05	0.10	2.9	11.4	35.8	49.3

Dividing the numerator and denominator of this expression by $\sum_i m_i$ (the total weight of the particles) and remembering that $m_i/\sum_i m_i = \eta_i$, we finally obtain

$$d_{10} = \frac{\sum_i (\eta_i/\bar{d}_i^2)}{\sum_i (\eta_i/\bar{d}_i^3)}. \quad (6)$$

In a similar way, Eqs. (3) and (4) lead to

$$d_{20} = \left(\frac{\sum_i (\eta_i/\bar{d}_i)}{\sum_i (\eta_i/\bar{d}_i^3)}\right)^{1/2}, \quad (7)$$

$$d_{30} = \left(\frac{1}{\sum_i (\eta_i/\bar{d}_i^3)}\right)^{1/3}. \quad (8)$$

Expressions (6)–(8) are more convenient for calculations, since η_i and $\bar{d}_i$ are derived directly from size-fraction weight data.

A characteristic parameter of any fractional particle-size distribution is the median diameter d_m (the diameter such that the total weight fraction of the particles falling between the lower diameter limit and this diameter is 0.5 [1, 4]). It is interesting to relate d_m to d_{10}, d_{20}, and d_{30}. However, no such relationship can be drawn directly from experimental particle-size distribution.

We will assume that the particle-size distribution is lognormal [1, 3]. This is indicated by experimental data for aerosols and suspensions [5, 6]. In this case, the particle-size frequency function is given by

$$\varphi(d) = \frac{100\log e}{\sqrt{2\pi}d\log\sigma}\exp\left[-\frac{(\log d - \log d_m)^2}{2\log^2\sigma}\right]. \quad (9)$$

In view of Eq. (9), the sums appearing in Eqs. (6)–(8) can be represented as integrals:

$$\sum_i \frac{\eta_i}{\bar{d}_i} = \int_0^\infty \frac{1}{x}\varphi(x)dx, \quad (10)$$

$$\sum_i \frac{\eta_i}{\bar{d}_i^2} = \int_0^\infty \frac{1}{x^2}\varphi(x)dx, \quad (11)$$

$$\sum_i \frac{\eta_i}{\bar{d}_i^3} = \int_0^\infty \frac{1}{x^3}\varphi(x)dx, \quad (12)$$

where x is the particle diameter. The integrals in Eqs. (10)–(12) are complete moments of order a for the lognormal distribution

$$M_a(\infty) = \int_0^\infty x^a\varphi(x)dx. \quad (13)$$

In formulas (10)–(12), $a = -1$, -2, and -3, respectively. By integrating Eq. (13) and designating the particle diameter by d, we obtain

$$M_a(\infty) = d_m^a \exp\frac{a^2\log\sigma}{2\log^2 e}. \quad (14)$$

By substituting the moments into Eqs. (6)–(8), we obtain, after rearrangements, expressions relating d_{10}, d_{20}, and d_{30} to the median diameter:

$$d_{10} = \frac{M_{-2}(\infty)}{M_{-3}(\infty)} = d_m\exp(-13.26\log^2\sigma), \quad (15)$$

$$d_{20} = \left(\frac{M_{-1}(\infty)}{M_{-3}(\infty)}\right)^{1/2} = d_m\exp(-10.60\log^2\sigma), \quad (16)$$

$$d_{30} = \left(\frac{1}{M_{-3}(\infty)}\right)^{1/3} = d_m\exp(-7.95\log^2\sigma). \quad (17)$$

Relationships (15)–(17) equable one to compute d_{10}, d_{20}, and d_{30} from experimental d_m and σ data. Since $\log^2\sigma > 0$, it follows from formulas (15)–(17) that $d_{30} > d_{20} > d_{10}$.

This method for deriving d_{10}, d_{20}, and d_{30} from the experimental fractional makeup is applicable not only to the lognormal but also any other particle-size distribution law.

By way of example, we will consider the particle-size composition of ash that results from the combustion of solid fuel (coal, turf) in a power plant [7]. Particle-size data for ash forming from different fuels (coal, turf, coal + turf) are necessary to optimize ash catching by the sprayed liquid in a Venturi scrubber and by the liquid film in a remote spray catcher. We studied particle-size distribution by liquid-phase sedimentation analysis [1] (table). Particle shape was determined by microscopy [2]. The particles were found to be plate-like, in agreement with data reported in [3, 4]. The mean diameter d_{20} was computed by formula (7). As the mean diameter of the ith fraction, $\bar{d}_i$, we took the arithmetic mean of the limiting diameters:

$$\bar{d}_i = (d_{i+1} + d_i)/2.$$

For ash forming from coal, turf, and a coal–turf mixture, d_{20} was calculated to be 14.2, 26.2, and 17.2 μm, respectively.

Taking into consideration particle shape in determining the characteristic particle size, as well as performing particle-size analysis by a method that is most similar in nature to the separation process of interest, will add to the reliability and accuracy of process design.

NOTATION

$\bar{d}_i$—mean particle diameter in the ith size fraction, m;

m_i—weight of the ith size fraction, kg;

n_i—number of particles in the ith size fraction;

η_i—weight fraction of the ith size fraction;

ρ—particle density, kg/m^3;

σ—standard deviation of the particle diameter from the mean value.

REFERENCES

1. Kouzov, P.A., *Osnovy analiza dispersnogo sostava promyshlennykh pylei i izmel'chennykh materialov* (Particle-Size Analysis of Industrial Dust and Ground Materials), Leningrad: Khimiya, 1987.
2. Gradus, L.Ya., *Rukovodstvo po dispersionnomu analizu metodom mikroskopii* (Manual of Microscopic Particle-Size Analysis), Moscow: Khimiya, 1979.
3. Reist, P., *Introduction to Aerosol Science*, New York: Macmillan, 1984.
4. *Spravochnik po pyle- i zoloulavlivaniyu* (Handbook of Dust and Ash Catching), Rusanov, A.A., Ed., Moscow: Energoatomizdat, 1983.
5. Levdanskii, E.I., Volk, A.M., and Plekhov, I.M., Particle-Size Distribution in Crushing, *Teor. Osn. Khim. Tekhnol.*, 1986, vol. 20, no. 5, p. 672.
6. Val'dberg, A.Yu. and Kirsanova, N.S., Calculating the Energy Efficiency of Mechanical Dust Catchers, *Teor. Osn. Khim. Tekhnol.*, 1992, vol. 26, no. 1, p. 145.
7. Zagoskina, N.V., Zykin, Yu.V., and Sokovnin, O.M., Particle-Size Analysis of Fly Ash from Power Plants, *Materialy IV Vavilovskikh chtenii* (Proc. IV Vavilov Workshop), Ioshkar-Ola: Mariiskii Tekh. Univ., 2000, part 3, p. 15.