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Efficiency of the Entrainment of Rodlike Particles by Rising Bubbles

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Abstract—The efficiency of the entrainment of rodlike particles by rising bubbles was determined experimentally. The observed and calculated data are in a satisfactory agreement.

The flotation rate is determined by the probability of a rising bubble colliding with a particle. This probability is characterized by particle-entrainment efficiency E, which is defined as the ratio of the number of particles that have actually been entrained by a bubble to the number of particles that could have been entrained if the flow lines did not deviate from the bubble surface (Fig. 1):

$$E = y_0^2/R^2. \quad (1)$$

In the case of spherical particles, the entrainment efficiencies for bubbles rising in a potential and in a Stokes flow are expressed as [1]

$$E_p = 3\frac{r}{R}, \quad (2)$$

$$E_s = \frac{3}{2}\left(\frac{r}{R}\right)^2. \quad (3)$$

In earlier publications, we reported expressions for E_p [2] and E_s [3] for rodlike particles:

$$E_p = \frac{27}{\pi}\left(\frac{l}{R}\right)^{3/2}, \quad (4)$$

$$E_s = \frac{4 + 13\sqrt{3}}{16\pi}\left(\frac{l}{R}\right)^{3/2} \approx 0.528\left(\frac{l}{R}\right)^{3/2}. \quad (5)$$

In the present paper, we report an experimental determination of E for rodlike particles and check the validity of formulas (4) and (5).

Our experiments were devoted to the flotation separation of a nutrient yeast suspension. The membranes of yeast cells are highly lyophobic owing to the adsorbed chitin (3.5–5.0% in terms of dry matter) [4]. The wedging pressure gradient Π in the liquid film of thickness h between the bubble and a yeast cell is positive:

$$\partial\Pi/\partial h > 0; \quad (6)$$

that is, the film separating the bubble and the cell tends to thinning and breaking with the formation of a three-phase wetted perimeter, which ensures sticking of the cell to the bubble surface [5]. Note that suspended yeast cells have the shape of a rod and are essentially uniform in size and morphology [6]. Because the cell membranes are thermolabile, the cell size was determined by light microscopy just at the inlet of the pilot flotation apparatus. The length and number of cells were determined as described in [7]. The smallest number of cells was 302. Figure 2 displays the size distribution of the cells.

The flotation concentration apparatus that was used in our experiments is a cylindrical vessel 720 mm in

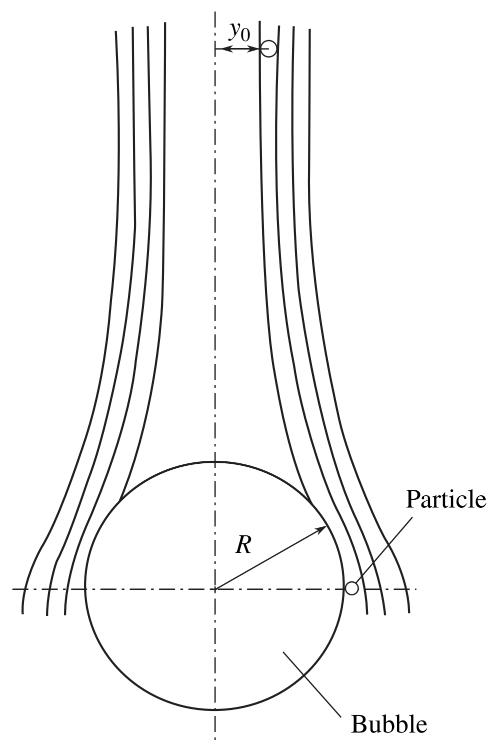

Fig. 1. Entrainment of a particle by a rising bubble.

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diameter, with a coaxial partition inside (Fig. 3). A gas-saturated yeast suspension is fed from a fermenter through the inlet pipe into the outer, annular zone of the flotator. This zone is divided into sections by radial plates which do not reach the bottom. Rising bubbles lift yeast cells. The concentrated suspension flows into the central bowl of the flotator and is withdrawn from there to be further concentrated. The yeast-depleted liquid from the outer zone flows below the radial plates toward the outlet pipe (which is perpendicular to the inlet pipe) and is discharged.

We operated the flotator in a steady-state mode and measured Q_i, φ_i, and C_i at the flotator inlet (i = 1), the outlet of the central bowl (i = 2), and the outlet of the outer zone (i = 3).

The flow rate was measured with a volumetric flowmeter. The gas content was measured by the volume–weight method [8], and the yeast concentration in the suspension was determined as described in [6]. To compare the data calculated by Eqs. (4) and (5) with the observed data, we estimated the bubble size by microphotography [9]. The observed data are presented in Table 1.

Q_i, φ_i, and C_i were determined from mass-balance equations. Since $\varphi_3 \approx 0$, we can write the following equations:

for the liquid phase,

$$Q_1(1-\varphi_1) = Q_2(1-\varphi_2) + Q_3; \quad (7)$$

for the solid phase (yeast cells),

$$Q_1C_1(1-\varphi_1) = Q_2C_2(1-\varphi_2) + Q_3C_1; \quad (8)$$

and for the gas phase,

$$Q_1\varphi_1 = Q_2\varphi_2. \quad (9)$$

The last equation disregards the spontaneous breaking of some bubbles during flotation; because of this, $Q_1\varphi_1$ was greater than $Q_2\varphi_2$ in all the experiments.

Since the bubble–particle associates were small and unstable, it was impossible to observe and count them directly and E was determined by an indirect method in terms of the flotation rate constant K_f [10]:

$$E = 4K_fRk_p/(3q). \quad (10)$$

The coefficient k_p, which characterizes the bubble-size distribution, was calculated by the formula $k_p = 1 + 2.32\sigma^2/d^2$ from bubble diameter data collected in three experiments (Table 2). Here, $d = \sum_j \bar{d}_j n_j/N$ is the mean bubble diameter, $\sigma^2 = \sum_j (\bar{d}_j - d)^2 n_j/N$ is the mean square deviation of $\bar{d}_j$ from d, $\bar{d}_j$ is the mean bubble diameter in the jth fraction, n_j is the number of bubbles in the jth fraction, and N is the total number of bubbles. From the data of Table 2, $k_p = 1.2$.

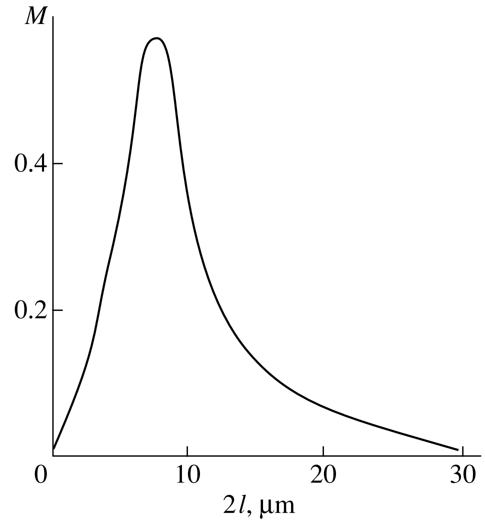

Fig. 2. Size distribution density of yeast cells.

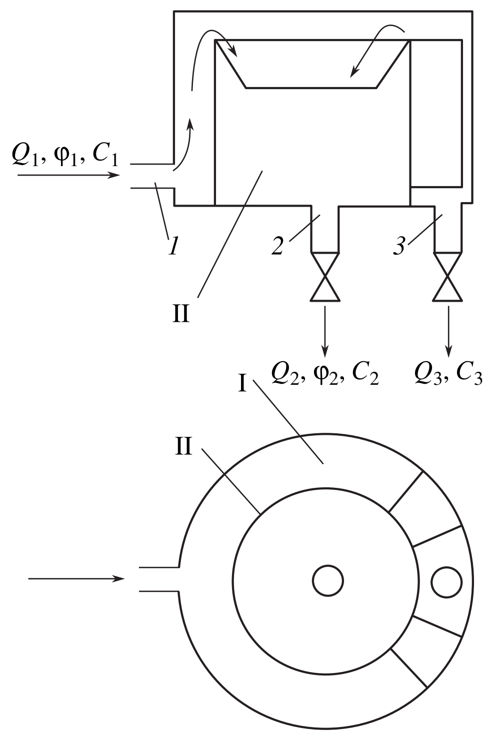

Fig. 3. Schematic of the flotator: (I) the outer (annular) zone, (II) the central bowl, and (*1–3*) sampling points.

The specific bubbling rate q, which appears in Eq. (9), was calculated as

$$q = Q_1\varphi_1/F. \quad (11)$$

The constant K_f was derived from the fundamental flotation equation [11]:

$$K_f = -\frac{1}{t}\ln\frac{C_3}{C_1}. \quad (12)$$

Table 1. Experimental data on the concentration of the yeast suspension

$R \times 10^4$, m	$Q_1 \times 10^4$, m^3/s	φ_1	C_1, kg/m^3	$Q_2 \times 10^5$, m^3/s	φ_2	C_2, kg/m^3	$Q_3 \times 10^5$, m^3/s	C_3, kg/m^3
1.2–1.4	2.60	0.75	35	6.0	0.60	92	3.6	1.8
1.3–1.5	3.00	0.80	35	7.0	0.66	86	3.6	2.0
1.3–1.5	3.20	0.81	36	7.4	0.73	98	3.5	2.2
1.4–1.6	3.30	0.83	40	7.6	0.75	110	3.8	2.9
1.5–1.7	3.70	0.85	38	8.2	0.76	109	4.0	3.3

$\varphi_3 \approx 0$, because the yeast-depleted liquid that leaves the outer zone of the flotator is virtually a single phase.

Table 2. Bubble-size distribution in the yeast foam

Δd, μm												N	d, μm	σ, μm	k_p
0–50	51–100	101–150	151–200	201–250	251–300	301–350	351–400	401–450	451–500	501–550	551–600				
–	–	2	18	174	50	14	–	–	–	19	2	279	260	84	1.24
–	–	3	12	140	121	42	–	16	–	6	4	344	270	71	1.16
–	–	–	19	104	132	49	27	14	–	12	7	364	290	82	1.18

The residence time of the yeast suspension in the flotator, t, was calculated as

$$t = V/Q_1. \tag{13}$$

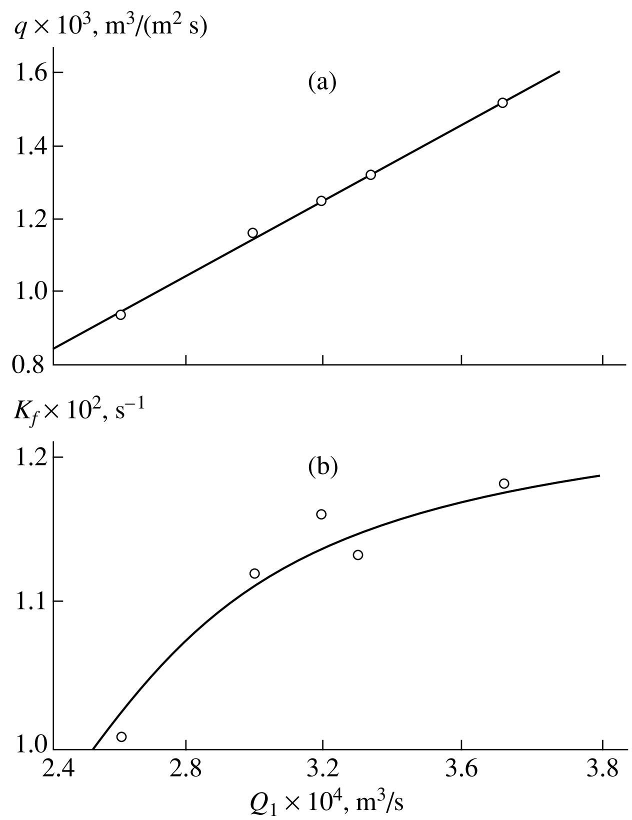

Fig. 4. (a) q and (b) K_f as functions of the flow rate of the yeast suspension.

Figure 4 plots the observed q and K_f data as functions of the flow rate of the yeast suspension, Q_1.

To plot theoretical E data for yeast cells, we preliminarily studied the flow dynamics of bubbles in the suspension. The rheological properties of the liquid phase of the suspension were studied by rotational viscometry [12] (Reotest-2 viscometer). The liquid phase was found to possess non-Newtonian properties. The flow curve of the liquid, which is shown in Fig. 5, was fitted to the power-law function

$$\tau = K\gamma^n \tag{14}$$

with $K = 9 \times 10^{-5}$ Pa s^n and $n = 1.31$.

The motion of bubbles in the system considered is characterized by the modified Reynolds number [12]:

$$\tilde{\mathrm{Re}} = \rho U^{2-n}(2R)^n/K. \tag{15}$$

Calculations have demonstrated that, at $\tilde{\mathrm{Re}} < 0.5$, the bubbles rise in a Stokes flow. For example, at $R = 0.15$ mm and $\varphi = 0.58$, $U = 0.60$ mm/s and $\tilde{\mathrm{Re}} = 0.19$. The rising velocity U of the bubbles in the non-Newtonian liquid was calculated by a formula taking into account the actual gas content of the liquid [13]:

$$U = \left[\frac{2\rho g}{3KI(3 + 2\varphi^{5/3})}\right]^{1/n} \times R^{(n+1)/n}(2 - 3\varphi^{1/3} + 3\varphi^{5/3} - 2\varphi^2), \tag{16}$$

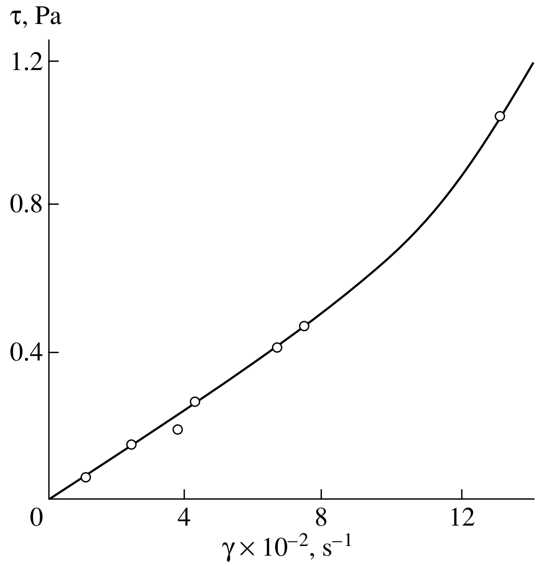

Fig. 5. The flow curve of the liquid phase of the yeast suspension.

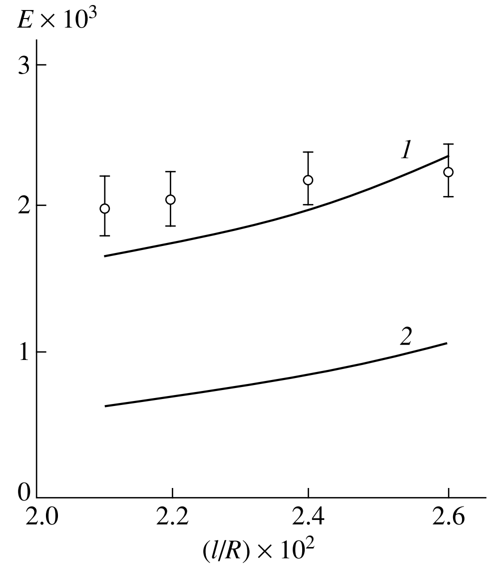

Fig. 6. E versus l/R as calculated by (*1*) Eq. (5) and (*2*) Eq. (3). The points represent the observed data.

where

$$I = \int_0^\pi \left[2\cos^2\theta\left(\frac{9}{4} - 5\varphi^{5/3} + 5\varphi^{10/3}\right) + \sin^2\theta(36 + 12\varphi^{5/3} + \varphi^{10/3})\right]^{(n-1)/2} \sin\theta d\theta,$$

and θ is the angular coordinate in a spherical coordinate system with the origin placed at the center of a rising bubble [14]. The integral I was calculated by a numerical method. At $n = 1$ (Newtonian liquid), we obtain $I = 2$ and formula (16) is identical to the expression derived [14] for the velocity of an ensemble of bubbles in a Newtonian liquid. At $n = 1$ and $\varphi = 0$, formula (16) reduces to the familiar Stokes formula for a single bubble:

$$U = 2\rho g R^2/(9\mu),$$

where μ is the dynamic viscosity of a Newtonian liquid.

The entrainment efficiency was calculated by formula (5), which takes into account that the lifted particles (yeast cells) are rodlike. In the calculation, we used the mean particle size $2l$, which was determined experimentally to be 7.1 μm (Fig. 2). The observed and calculated data are presented in Fig. 6. For comparison, Fig. 6 plots the data calculated by formula (3) for spherical particles of the same size (curve *2*). As is seen from Fig. 6, the observed and calculated data as a whole agree satisfactorily and the closest agreement between the theory and experiment is observed at $l/R = 0.024$–0.026 or $R = 130$–150 μm. It is in this R range that most of the particles (75%) fall. However, formula (3) yields underestimated E data, with a deviation from the observed data above 65%. Obviously, this result is unacceptable and suggests that a correction factor should be introduced in formula (3). Our experimental data as a whole validate formulas (4) and (5) for nonspherical particles.

ACKNOWLEDGMENTS

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NOTATION

C—concentration, kg/m^3;
d—diameter of a gas bubble, m;
E—entrainment efficiency;
F—flotation area, m^2;
g—acceleration of gravity, m/s^2;
K—rheological constant, Pa s^n;
K_f—flotation rate constant, s^{-1};
k_p—polydispersity coefficient;
$2l$—length of a rodlike particle, m;
M—number of yeast cells;
n—rheological constant;
Q—flow rate, m^3/s;
q—specific bubbling rate, m^3/(m^2 s);
R—bubble radius, m;
r—radius of an entrained particle, m;
t—flotation time, s;
U—velocity of a rising gas bubble, m/s;
V—volume of the flotation zone, m^3;

y_0—distance (at infinity) between the flow axis and the farthest particle trajectory contacting the bubble (Fig. 1), m;

γ—shear velocity, s^{-1};

μ—dynamic viscosity, Pa s;

ρ—density of the liquid, kg/m^3;

τ—shear stress, Pa;

φ—gas content.

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