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Measurement of aqueous foam rheology by acoustic levitation

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An experimental technique is demonstrated for acoustically levitating aqueous foam drops and exciting their spheroidal modes. This allows fundamental studies of foam-drop dynamics that provide an alternative means of estimating the viscoelastic properties of the foam. One unique advantage of the technique is the lack of interactions between the foam and container surfaces, which must be accounted for in other techniques. Results are presented in which a foam drop with gas volume fraction $\phi = 0.77$ is levitated at 30 kHz and excited into its first quadrupole resonance at 63 ± 3 Hz. By modeling the drop as an elastic sphere, the shear modulus of the foam was estimated at 75 ± 3 Pa.

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I. INTRODUCTION

Aqueous foams, which consist of gas bubbles suspended in liquids, are important in a variety of industrial applications, as described in Ref. [1], including firefighting, oil recovery, and drilling. Effective transport and use of wet foams, as well as the conception of new uses, requires a fundamental understanding of their macroscopic response to applied stress or strain fields. A number of theoretical and experimental studies indicate that foams behave as linear viscoelastic solids at low applied stresses. These studies are reviewed in various texts [2–4] and in the review paper by Weaire and Fortes [5].

This Rapid Communication concerns the dynamics and viscoelastic properties of aqueous foam drops, in which each drop contains many bubbles. An experimental technique is described in which a foam drop is acoustically levitated in air and acoustically excited into oscillatory motion. Acoustic levitation provides a noncontact means of suspending the foam during testing, so that the boundaries of the foam are not disturbed by container walls. By measuring the oscillatory motion of the drop, the viscoelastic properties of the foam may be estimated. The technique will eventually enable the study of the continuum of states for the range $0 < \phi < 1$. One critical experimental hurdle to be overcome in experimentally investigating the entire ϕ range is the transient coarsening of the foam [6], in which gravity drains the liquid to the bottom of the sample, and diffusion causes large bubbles to grow at the expense of smaller ones. This difficulty can be partially overcome by using the technique described here in a microgravity environment.

Initial experimental results are presented in which an aqueous foam drop was levitated and acoustically forced into one resonance. The resonance frequency and mode shape were recorded. An analytical model of the experiment was developed by approximating the drop geometry as a sphere. The shear modulus was estimated by adjusting its value in the analytical model to match the experimentally determined natural frequency. As the Poisson's ratio of the foam was not measured, these calculations were performed over the expected range of Poisson ratio values, from 0 to $\frac{1}{2}$. The authors are aware that certain classes of foams have negative Poisson ratios; however, such behavior was not observed in these experiments.

Previous experimental investigations have generally been restricted to either dilatational or shear strain of the foam. Quasistatic shear strain techniques have been developed based on measuring the oscillations of objects embedded in the foam sample [7,8]. Other quasistatic shear strain techniques have been based on either concentric-cylinder viscometers [9] or parallel plate rheometers [10]. The latter two techniques have been extended to measure dynamic shear modulus [11–13]. Dynamic shear modulus was measured by propagating shear waves in a foam sample [14] and dynamic dilatational modulus was measured by propagating dilatational waves through a foam sample [15]. Measured values of the shear modulus are summarized in Table I and are compared below to values from our technique.

In the context of these previous experimental techniques for estimating viscoelastic parameters, our technique offers three advantages. First, it provides a noncontact means of supporting the sample. This is advantageous because the boundaries of the foam are not altered by a container wall. Second, it allows one to test small samples of foam containing a small number of bubbles. This capability should allow one to validate theoretical models, such as the one recently proposed by Durian [16], based on the interactions of a small number of neighboring bubbles. Third, it allows one to simultaneously measure shear and compressional parameters, whereas other techniques are restricted to either of these.

TABLE I. Summary of shear modulus measurements.

Reference	Void fraction	Frequency (Hz)	Shear modulus (Pa)
[6]	0.926–0.936	0.01–10	100–500
[8]	0.93–0.941	0	168-251
[9]	0.75–0.976	0	61–351
[11]	0.92–0.97	0.06–1.6	50–100
[12]	Not reported	0.08–16	20–40
[13]	0.7–0.94	0–16	10–200
[13]	0.5–1	0.16	0–160

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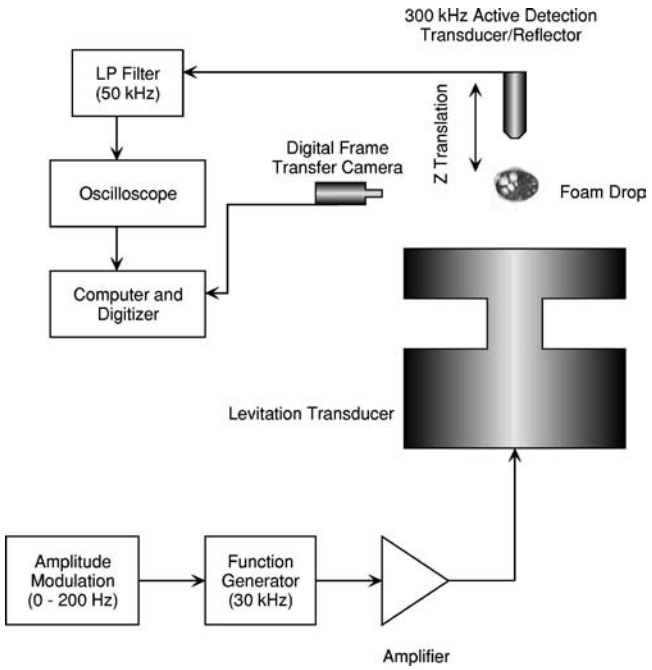

FIG. 1. Experimental apparatus.

II. EXPERIMENTAL TECHNIQUE

A schematic of the foam drop levitator is shown in Fig. 1. This apparatus is similar to that used by Marston and Apfel [17], Tian, Holt, and Apfel [18], and Trinh, Marston, and Roby [19] to levitate liquid drops and estimate their rheological parameters. The foam drop is acoustically levitated in a standing wave pressure field in the air gap between the horn and reflector. The acoustic pressure is varied sinusoidally at 30 kHz. Nonlinear radiation pressure serves to hold the foam in a fixed position slightly below a pressure node, where the buoyant force is balanced [20]. In practice, this limits the upper range of foam drop radii to 0.5 cm at an operating frequency of 30 kHz, and correspondingly larger radii at lower frequencies. The acoustic pressure produced by the horn is approximately symmetric about a vertical axis passing through the foam drop.

The foam drop is created by first levitating a drop of liquid and then injecting air into the drop using a hypodermic needle. For the results presented here, the foam liquid was composed of water with 11% glycerol and 0.1% (10CMC) Triton-X-100 surfactant by weight. The acoustic levitation pressure is modulated at a frequency that is orders of magnitude lower than the levitation frequency of 30 kHz. For the results presented below, the frequency of the excitation pressure was 63 ± 3 Hz. By sweeping the modulation frequency and identifying local maxima in the drop oscillation response amplitude, the eigenmodes of the foam drop are excited.

III. RESULTS

In this and the following section, initial experimental results are presented which demonstrate that a foam drop may be levitated and resonated in a mode shape that approximates one of the modes of an elastic sphere. Following the technique described in the preceding section, a foam drop with an effective radius of 3.78 mm was levitated, as shown in the photograph on the left-hand side of Fig. 2. Next, the acoustic pressure was modulated until the global response of the foam reached a local maximum, which occurred at 63 ± 3 Hz. A photograph of the foam drop oscillating at this modulation frequency is shown on the right-hand side of Fig. 2.

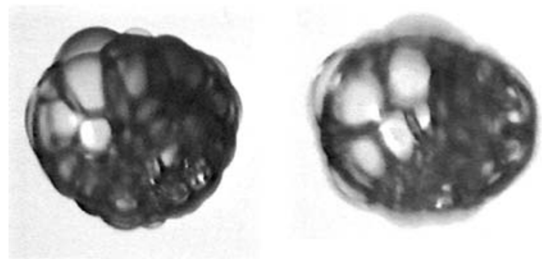

FIG. 2. Foam drop levitated in an acoustic field. The drop is shown without modulation (left) and with modulation (right). The blurred region surrounding the modulated drop represents the range of outer surface vibration, as the shutter speed of the camera was much less than the period of vibration.

The entire drop was then captured between two glass plates of known spacing and the resulting squeezed monolayer image was analyzed to determine the void fraction and bubble size distribution. The void fraction was found to be $\phi=0.77\pm0.05$ and the individual bubble volumes varied from $0.1-1.2\ \text{mm}^3$, with most of the volumes falling in the range $0.1-0.5\ \text{mm}^3$. Based on these measurements and the density of the foaming liquid, the density of the foam was estimated at 231 kg/m^3.

IV. THEORETICAL MODEL FOR VISCOELASTIC PARAMETER ESTIMATION

A theoretical model is necessary to relate the observed modal information to the viscoelastic properties of the foam drop. In this section, a theoretical model of the foam is developed that approximates the foam drop when four assumptions are satisfied. First, the shape of the foam drop is approximated as spherical, which is somewhat justified by Fig. 2. While a wide body of literature exists on the effects of nonsphericity in liquid drop vibrations, the analogous effects on viscoelastic drop vibrations are not well understood. Accounting for these effects will be addressed in a future study.

Second, dissipation is neglected. Depending on the nature of any actual damping in the foam, the observed resonance frequency would be slightly altered. Third, the foam vibrations are assumed linear. In the experiment, the vibrations of the foam were made as linear as possible by using the minimum modulation amplitude necessary to observe the resonance and the mode shape. Fourth, it was assumed that the observed resonance of the foam drop was not altered by interactions with the surrounding air. This assumption will be justified later by showing that the derived acoustic impedance of the foam drop is orders of magnitude larger than the surrounding air.

Based on these assumptions, the foam drop was modeled as an elastic sphere. The modes of elastic spheres were first analyzed by Lamb [21], while detailed numerical analyses of the characteristic equations were presented by Satô and Usami [22]. The modes are classified as either torsional or spheroidal. In the experiment, only one spheroidal mode was observed. Using the notation of Sato and Usami, the displacement field of an elastic sphere vibrating in a spheroidal mode is

$$u = A_{mn} U(r) P_n^m(\cos\theta)\cos(m\phi), \tag{1}$$

$$v = A_{mn} V(r) \frac{d}{d\theta} P_n^m(\cos\theta)\cos(m\phi), \tag{2}$$

$$w = -mA_{mn} V(r) \frac{P_n^m(\cos\theta)}{\sin\theta}\sin(m\phi), \tag{3}$$

where u, v, and w are the radial, colatitudinal, and azimuthal components of displacement. The azimuthal angle θ is measured in the plane of the photograph in Fig. 2. The A_{mn} is a modal amplitude and $U(r)$ and $V(r)$ are complicated functions of the radial coordinate. These functions are given by Sato and Usami and are unimportant to the experiment described in the preceding section, as radial variations were not observable.

During the experiment, no displacement variations were observed in the polar angle, ϕ, so that the polar mode number was determined to be $m=0$. From Fig. 2, one observes two periods of oscillation in the azimuthal angle, θ. By using the definition $P_2^0(x) = (1/4)[3\cos(2\theta)+1]$, the azimuthal mode number is identified as $n=2$.

The natural frequencies of the spheroidal modes satisfy the characteristic equation $C=0$, where C is given by [22]

$$C = \frac{2\xi}{\eta}\left[\frac{1}{\eta} + \frac{(n-1)(n+2)}{\eta^2}\left(\frac{J_{(n+3/2)}(\eta)}{J_{(n+1/2)}(\eta)} - \frac{n+1}{\eta}\right)\right] \times J_{(n+3/2)}(\xi) + \left[-\frac{1}{2} + \frac{(n-1)(2n+1)}{\eta^2} + \frac{1}{\eta}\left(1 - \frac{2n(n-1)(n+2)}{\eta^2}\right)\frac{J_{(n+3/2)}(\eta)}{J_{(n+1/2)}(\eta)}\right] J_{(n+1/2)}(\xi). \tag{4}$$

The normalized dilatational and shear wavenumbers are

$$\xi = ha = \omega a / c_d, \tag{5}$$

$$\eta = ka = \omega a / c_s, \tag{6}$$

where $c_d = \sqrt{E/(\rho(1-\nu^2)}$ and $c_s = \sqrt{G/\rho}$ are the dilatational and shear wave speeds and a is the sphere radius.

Although researchers have previously treated foams as incompressible, corresponding to a Poisson's ratio of $\frac{1}{2}$, the relatively low void fraction of the foam used in the experiment left some uncertainty in the Poisson's ratio. Choosing G and ν as the independent elastic parameters, the natural frequencies of the $m=0$ and $n=2$ mode are indicated by values of ω that result in $C=0$ for particular values of G and ν. Note that E is related to these two parameters by $E = 2G(1+\nu)$.

To find the natural frequencies, ξ was taken as a nondimensional frequency and varied for all possible values of ν. The characteristic function C was computed at each value of ν and ξ and the zero values indicated the natural frequencies.

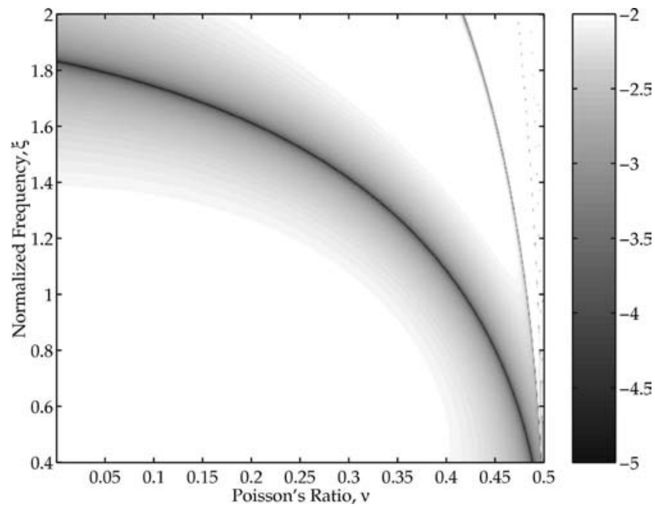

FIG. 3. Contour plot of the logarithm of the characteristic function, $\log_{10}(|C|)$, for the $n=2$ spheroidal mode of an elastic sphere. Dark contours corresponding to low values of C represent the natural frequencies of the sphere.

A contour plot of C versus ν and ξ is shown in Fig. 3. In this figure, the fundamental mode appears as a dark line, beginning at $\xi \approx 1.85$ for $\nu=0$ and decaying to $\xi \approx 0.6$ at $\nu = 0.47$. Other modes appear at high values of ν; however, the fundamental quadrupole mode was determined to be the one excited in the experiment by visual inspection. If indeed the foam is incompressible, then modal techniques that seek to characterize particular spheroidal modes will be ineffective due to high modal overlap in the range $\nu > 0.47$.

For clarity, the dark line in Fig. 3 was extracted and plotted in Fig. 4. Based on the fact that the fundamental mode was experimentally observed to have a resonance frequency of 63 ± 3 Hz, the shear modulus of the foam was computed according to

$$G = \rho\left(\frac{\omega a}{\xi}\right)^2 \frac{1-2\nu}{2(1-\nu)}, \tag{7}$$

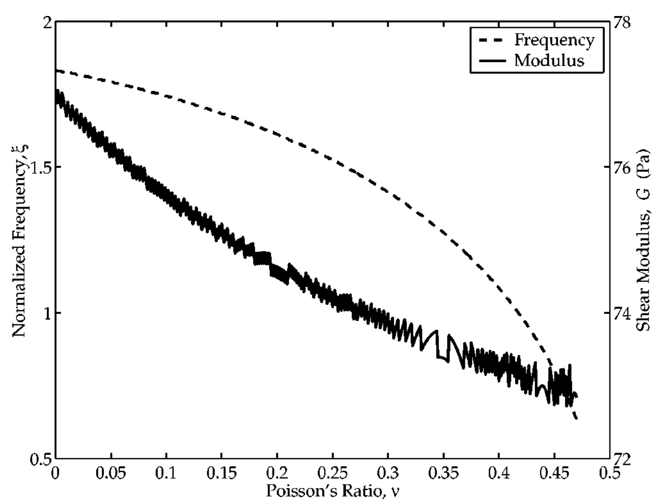

FIG. 4. Plot of the $n=2$ fundamental spheroidal frequency of an elastic sphere, obtained by extracting the lower contour of Fig. 3. Plotted against the right-hand ordinate is the corresponding shear modulus of a foam drop that would resonate at 63 Hz.

where the frequency was $\omega = 2\pi(63)$ and ξ was taken as the curve shown in Fig. 4. The corresponding values of G are also shown in Fig. 4 plotted against the right-hand axis. The shear modulus is relatively insensitive to variations in Poisson's ratio, varying only about 5% over the range $0 < \nu < 0.47$. These values of shear modulus fall in the range of values summarized in Table I. In particular, Saint-Jalmes and Durian (see Fig. 4 in [13]) report a shear modulus value for a foam with the same void fraction, excited at 0.16 Hz, that agrees with the modulus range shown in Fig. 4 to within 20%. Such close agreement is intriguing, given the differences in foam composition as well as excitation type and frequency.

V. CONCLUSIONS

This work has demonstrated that foam drops can be effectively levitated in air and forced into resonance by modulating the levitation pressure. An approximate analytical model has been used to relate the resonance frequency to the shear modulus of the foam. This derived modulus was found to be be relatively insensitive to Poisson's ratio, varying less than 5% over the range $0 < \nu < 0.49$, and in agreement with other experimental techniques. Future work will seek to confirm these initial findings by exciting multiple resonances and independently measuring the Poisson's ratio, as one means of investigating the long held assumption of incompressibility.

In the long term, we hope to incorporate a unique theoretical approach for the wet limit and merge the wet and dry descriptions to form a global picture of the physics and rheology of foams for arbitrary void fraction. The inclusion of compressibility in describing the acoustic and rheological problem will be a key component in any such undertaking. We hope to refine our experimental technique to include the ability to control and vary the ambient pressure as well as the acoustic pressure. Finally, we hope to investigate especially the wet and critical void fraction limits in a reduced-gravity environment, so that buoyancy and coarsening do not destroy the foam's character.

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